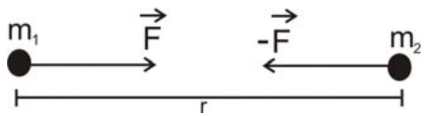
Chapter 2

GRAVITATION

NEWTON'S LAW OF GRAVITATION:



Statement: Every particle in the universe attracts every other particle with a force which is directly proportional to the product of the masses and inversely proportional to the square of the distance between them and this force acts along the line joining the two particles and independent of the presence of other bodies.

Gravitational force is a central force. m_2 attracts m_1 with a gravitational force $\vec{F}$ directed towards m_2, m_1 attracts m_2 with a force $-\vec{F}$ towards m_1. The force $\vec{F}$ and $-\vec{F}$ form action reaction pair, equal in magnitude and opposite in direction.

$$F \propto m_1 . m_2$$

$$F \propto \frac{1}{r^2}$$

$$F = \frac{Gm_1m_2}{r^2} \quad \text{-------------------} \ (1)$$

Where, G is constant of proportionality known as **universal gravitational constant** its

Value is **6.67×10^{-11} Nm^2/kg^2.**

In vector form it is written as

$$\vec{F}_{21} = \frac{Gm_1m_2}{r^2}\hat{r}_{12}$$

Where $\hat{r}_{12}$ is the unit vector from m_1 to m_2.

Dimensions of G:

$$G = \frac{F . r^2}{m_1 . m_2}$$

$$G = \frac{[M^1L^1T^{-2}][M^0L^2T^0]}{[M^2L^0T^0]}$$

$$G = [M^{-1}L^3T^{-2}]$$

Definition of G: From (1) we have,

$$G = \frac{F . r^2}{m_1 . m_2}$$

If $r = 1$, $m_1 = m_2 = 1$,

$$G = F$$

Hence,

It is defined as the magnitude of force of attraction between two bodies each of unit mass and separated by a unit distance.

RELATION BETWEEN 'G' AND 'g' (ACCELERATION DUE TO GRAVITY):

Let g be the acceleration due to gravity on the earth's surface . Let M be the mass of the earth of radius R and m be the mass of the body.

On the surface the surface of earth:

Weight of the body = gravitational force of attraction.

$$mg = \frac{G.Mm}{R^2}$$

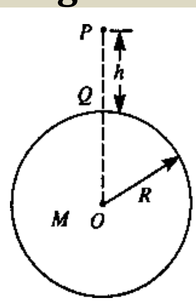
$$g = \frac{GM}{R^2}$$

$$GM = g R^2$$

VARIATIONS IN THE VALUE OF g:

The value of acceleration due to gravity *(g)* varies as we go above or below the surface of the earth. It also varies from place to place on the surface of the earth.

Variation of g with altitude:



Consider earth to be a sphere of radius *R* and mass M.

The acceleration due to gravity on the surface of earth (point *Q* in Fig.) is

$$g = \frac{\mathrm{GM}}{\mathrm{R}^2} \text{ ---------- } (1)$$

Consider a point *P* at a height *h* above the surface of the earth. The acceleration due to gravity at point *P* is-

$$g_h = \frac{\mathrm{GM}}{(\mathrm{R}+\mathrm{h})^2} \text{ --------- } (2)$$

Dividing (2) by (1)

$$\frac{g_h}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} \text{ --------- } (3)$$

$$g_h = \frac{R^2}{(R+h)^2} g$$

$$\therefore \quad \mathbf{g_h < g}$$

Thus, as we go above the earth's surface acceleration due to gravity goes on decreasing.

$$\frac{g_h}{g} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g_h}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g_h}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

$\because$ h < < < R higher powers of h/ R can be neglected

$\therefore$ Using Binomial Theorem,

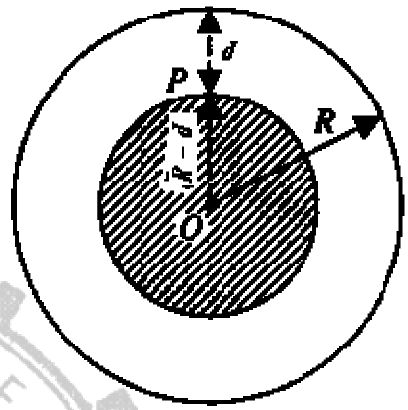
$$\frac{g_h}{g} = \left(1-\frac{2h}{R}\right)$$

$$g_{\mathbf{h}} = \left(1-\frac{2h}{R}\right)\mathbf{g} \text{ --------- } (4)$$

Using eq (4) value of acceleration due to gravity can be determined when h is small as compared to R.

Variation of *g* with depth.

Consider the earth to be a sphere of radius *R* and mass *M.*



The acceleration due to gravity at the surface of the earth is

$$g = \frac{\mathbf{GM}}{\mathbf{R}^2} \text{ ----- } (1)$$

If ρ is density of the earth, then,

$$\text{Density } (\rho) = \frac{\textbf{mass(M)}}{\textbf{Volume(V)}}$$

$$\therefore M = \rho V$$

$$\text{But, } V = \frac{4}{3}\pi R^3$$

$$\therefore M = \frac{4}{3}\pi R^3 \rho$$

Thus, substituting in eq.(1) acceleration due to gravity in terms of density is given by-

$$g = \frac{\mathbf{G}}{\mathbf{R}^2} \times \frac{4}{3}\pi R^3 \rho$$

$$g = \frac{4}{3}\pi R \rho G \text{ -------- } (2)$$

Consider a point P which is in side the earth below the earth's surface at depth *d*. Its distance from point O is (R-d).

A body at point P will experience force only due to the portion of earth of radius (R-d).

The outer spherical shell, whose thickness is d, will not exert any force on the body at point P. Let M be the mass of the earth of portion of radius (R-d) then –

$$g_d = \frac{GM'}{(R-d)^2} \text{ ---------- (3)}$$

But, M' = $\frac{4}{3}\pi(R-d)^3\rho$

$$\therefore g_d = \frac{G}{(R-d)^2}\frac{4}{3}\pi(R-d)^3\rho$$

$$\therefore g_d = \frac{4}{3}\pi(R-d)G\rho \text{ -------- (4)}$$

Dividing equation (4) by (2)

$$\frac{g_d}{g} = \frac{R-d}{R}$$

$$\therefore g_d = \left(1-\frac{d}{R}\right)g \text{ --------- (5)}$$

Therefore, the value of acceleration due to gravity decreases with depth.

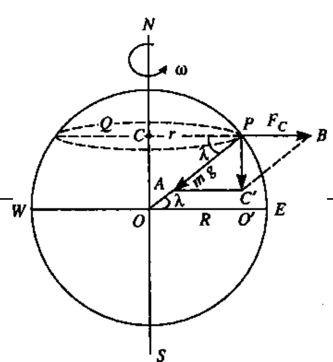
The acceleration due to gravity at the centre of earth can be found by substituting $d = R$ in equation (5).

$$\therefore g_d = \left(1-\frac{R}{R}\right)g$$

$$\therefore g_{centre} = 0$$

Variation in the value of g due to rotation of the earth :

Earth is rotating about its own axis at an angular velocity ω (= one revolution per 24 hours). The line joining the north and south poles is the axis of rotation. As a result of rotation, every point on the earth moves along a circular path with the same angular velocity ω. A point at the equator moves in a circle of radius equal to the radius of earth and the centre of the circle is the same as the centre of the earth. For any other point on the earth, the circle of rotation is smaller than this.



Consider earth to be a sphere of mass *M* and radius *R*. Suppose a particle of mass m is situated at point P on the surface of the earth as shown in figure.

Let λ be the latitude of the point i.e. $\angle$POE = λ. Suppose g is the acceleration due to gravity in the absence of rotational motion of the earth. In that case, the particle at P would have been attracted toward the centre O of the earth.

Therefore true weight mg of the particle is directed toward O and is represented by the vector $\overrightarrow{PA}$.

Due to rotation of earth, let, angular speed of earth is ω the particle at P moves along a circular path whose centre is C and radius r (=CP). Since $\angle$OPC = λ, r = R cos λ.

The centrifugal force on the particle due To rotational motion of the earth acts along the radius of the circular path in outward direction. The magnitude of centrifugal force is given by;

$$F_C = mr\omega^2 = mR\omega^2\cos\lambda$$

The centrifugal force is represented by the vector $\overrightarrow{PB}$. The apparent The apparent weight mg' of the particle is equal to the resultant of actual weight (=mg) and the centrifugal force $F_C (= mR\omega^2\cos\lambda)$.

Complete the parallelogram PAC'B. Then diagonal PC' of the parallelogram represents the apparent weight mg' of the particle.

Now,

$$PC'^2 = PA^2 + PB^2 + 2PA\times PB\times COS(180^0-\lambda)$$

$$\therefore (mg')^2 = (mg)^2 + (mR\omega^2\cos\lambda)^2 + 2\times mg\times mR\omega^2\cos\lambda(-\cos\lambda)$$

Or

$$m^2(g')^2 = m^2(g)^2 + m^2(R\omega^2\cos\lambda)^2 - 2m^2(gR\omega^2\cos^2\lambda)$$

$$(g')^2 = g^2 + R^2\omega^4\cos^2\lambda - 2gR\omega^2\cos^2\lambda$$

$$(g')^2 = g^2\left[1+\frac{R^2\omega^4\omega s^2\lambda}{g^2} - \frac{2R^2\omega^2\cos^2\lambda}{g}\right]$$

$$g' = g\left[1+\frac{R^2\omega^4\cos^2\lambda}{g^2} - \frac{2R\omega^2\cos^2\lambda}{g}\right]^{1/2}$$

Now, the numerical value of $\frac{R\omega^2}{g}$ is very small. Hence, $R^2\omega^4/g^2$ will be still smaller. Therefore, neglecting the factor containing $R^2\omega^4/g^2$, we get,

$$g' = g\left[1 - \frac{2R\omega^2\cos^2\lambda}{g}\right]^{1/2}$$

$$= g\left[1 - \frac{1}{2}\times\frac{2R\omega^2\cos^2\lambda}{g} + \textbf{higher terms}\right]$$

Neglecting the terms containing higher powers of $R\omega^2\cos^2\lambda/g$, we get,

$$g' = g\left[1 - \frac{R\omega^2\cos^2\lambda}{g}\right]$$

$$g' = g - R\omega^2\cos^2\lambda \text{ -------- } (1)$$

From equation (1), it is clear that acceleration due to gravity

a) decreases on account of rotation of the earth

b) increases with the increase in the latitude of the place ($\because$ cos λ decreases as λ increases). This means that value of g increases as we go from equator to the poles.

At equator: At equator $\lambda = 0^0$
so that cos λ = cos 0^0 = 1

$\therefore g' = g - R\omega^2$ ----------- (2)

Therefore, value of acceleration due to gravity is minimum at the equator. This is expected because the particle at the equator executes a circle of maximum radius. Therefore, the centrifugal force is maximum.

At poles: At poles, $\lambda = 90^0$
so that cos λ = cos 90^0 = 0

$\therefore g' = g$ ……. Maximum

Hence the value of g is maximum at the poles. This is expected because the particle at the pole moves in a circle of zero radius. Therefore, no centrifugal force acts on the particle.

Definition of Escape velocity (V_e) :

Maximum vertical velocity required to take the satellite just out side the earth's gravitational influence is called as escape velocity.

If vertical velocity is less than escape velocity, body falls on the earth's surface but if vertical velocity is greater than or equal to escape velocity body goes out of earth's gravitational influence permanently.

Definition of critical velocity / orbital velocity (V_c) :

It is the horizontal velocity imparted to the satellite so that the satellite orbits around the earth in a stable circular orbit with constant magnitude of this velocity.

Satellite:

A body which orbits in a closed orbit around another larger body, like planets, under the gravitational influence are called **satellites.**

Moon is a natural satellite of earth, INSAT 3 B is an artificial satellite of the earth.

Q.Why minimum of two stage rocket is used for the projection of satellite?

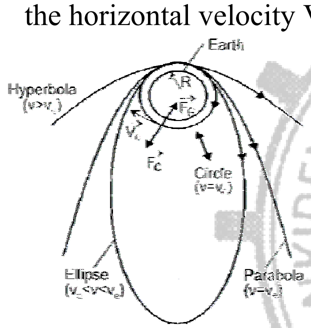
For a single stage rocket carrying satellite, if the velocity of projection is less than escape velocity, the satellite will come back to the earth, and if the velocity of projection is greater than or equal to the escape velocity, the satellite will escape earth's gravitational influence. Hence, is not possible to put a satellite into earth's orbit using a single stage rocket. Hence minimum of two stage rocket is used. The first stage is used to carry the satellite to a certain height the second stage and the satellite is rotated through 90^0 and with the help of second stage it is projected in the horizontal direction to put the satellite in a stable circular orbit.

PROJECTION / LAUNCHING OF A SATELLITE:

The satellite is held at the tip of rocket of minimum of two stages. The fuel of the first stage is ignited. The **up thrust** takes the rocket vertically to a certain height and the first stage gets detached. By guidance and tracking system, the second stage and satellite is rotated through 90^0. This second stage is used to give the horizontal velocity to the satellite and finally gets detached. The satellite alone starts orbiting around the earth in a pre-determined orbit.

DIFFERENT CASES OF SATELITE MOTION:

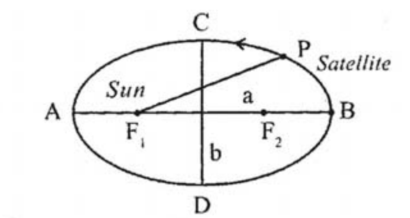
The nature of the path of satellite depends on the horizontal velocity V_h.



Case I: If the horizontal velocity is less than a certain velocity called critical velocity, the satellite falls to the earth after following a **spiral path** and falls on the earth.
Case II: If horizontal velocity is slightly less than critical velocity then it follows elliptical path with point of projection at apogee.
Case III: If the horizontal velocity equals the critical velocity, the satellite orbits in a stable **circular orbit.**
Case IV: If the horizontal velocity is greater than critical velocity but less than escape velocity, the satellite orbits around the earth in a **elliptical orbit** with point of projection at perigee.
Case V: If the horizontal velocity equals the escape, velocity the satellite will escape from the earth's gravitational field and be lost in space after describing a **parabola.**
Case V: If the horizontal velocity is greater than escape velocity the satellite will escape from the earth's gravitational field after describing a **hyperbola.**

Keplar's Laws

FIRST LAW: LAW OF ORBIT

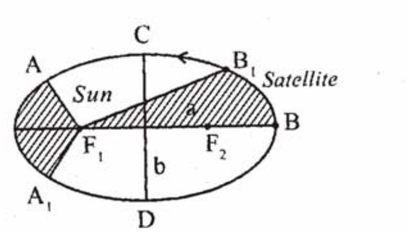


Statement: Each planet revolves around the sun in an elliptical orbit with the sun as one of the foci. Planet (P) revolves in an elliptical orbit as shown in fig with sun (S) at the focus F_1. When the planet reaches at A it is nearest to sun and this minimum distance (AF_1) is called perigee. When it is at B, it is at farthest distance from the sun. The distance BF1 is known as apogee. AB and CD are major and minor axes of ellipse.

SECOND LAW: LAW OF AREA

Statement: The position vector of planet from the sun sweeps out equal area in equal time of the area velocity of the planet around the sun always remains constant.

Suppose that at certain time, planet is at B and after time dt it moves to B_1. Then position vector of planet sweeps out area BF_1B_1 in time dt. Again in time δt the planet moves from A to A_1 and sweeps out area AF_1A_1 then according to second law, areas AF_1A_1 and BF_1B_1 must be same or equal. If dA is area swept out in dt then areal velocity is defined as $\frac{dA}{dt}$ and according to second law $\frac{dA}{dt}$ is constant.



THIRD LAW: LAWS OF PERIOD

The square of the period of any planet

around the sun is directly proportional to the cube of the semi major axis of the elliptical orbit.

If T is the time period of the planet and a is the semi major axis then

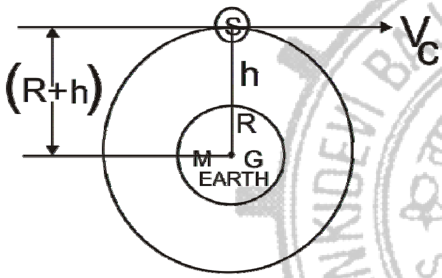
$$T^2 \propto a^3$$

If T_1 and T_2 are the periods of any two planets and a1 and a2are their semi major axes then

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

Q.OBTAIN AN EXPRESSION FOR CRITICAL VELOCITY OR ORBITAL VELOCITY.

Consider a satellite of mass ‘m’ raised to a height h’ above the earth’s surface. It is projected with critical velocity V_c.



The satellite starts orbiting in a stable circular orbit of radius (R+h) where R is the radius of the earth. Let M be the mass of the earth.

For a stable circular orbit,

Centripetal force = gravitational force of attraction.

$$\frac{m\, v_c^2}{(R+h)} = \frac{G.Mm}{(R+h)^2}$$

$$v_c^2 = \frac{GM}{R+h}$$

$$v_c = \sqrt{\frac{GM}{(R+h)}} \quad \text{------- (1)}$$

As there is no, mass term, critical velocity is independent of the mass of the satellite and as ‘GM’ is constant critical velocity is inversely proportional to the square root of the radius of the orbit.

If the satellite is orbiting close to the earth.
h << R then

$$R + h \approx R$$

$$\therefore v_c = \sqrt{\frac{GM}{R}}$$

Substituting GM = g R^2 in above equation

$$v_c = \sqrt{\frac{g\,R^2}{R}}$$

$$v_c = \sqrt{gR} \quad \text{------- (2)}$$

$$\mathbf{v_c = 7.9\ km/s}$$

This is the expression for critical velocity of satellite orbiting close to earth.

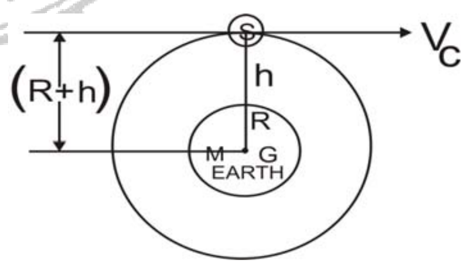
Substituting GM = g_h (R + h)^2 in eqn. (1)

$$v_c = \sqrt{\frac{g_h (R+h)^2}{R+h}}$$

$$v_c = \sqrt{g_h (R+h)} \quad \text{--------- (3)}$$

This is an expression for critical velocity in terms of acceleration due to gravity at a height h.

Q. Obtain an expression for time period a satellite. Show that the square of the period of satellite is directly proportional to the cube of the radius of the orbit.



Consider a satellite of mass ‘m’ raised to a height h above the earth’s surface. It is projected with a critical velocity V_c. The satellite starts orbiting in a stable circular orbit of radius (R + h), where R is the radius of the earth. Let M be the mass of the earth.

Time period is the time taken by the satellite to complete one rotation around the earth.

Time = distance / velocity

$$\textbf{Time period} = \frac{\textbf{circumference of circular orbit}}{\textbf{critical velocity}}$$

$$T = \frac{2\pi(R+h)}{V_c}$$

$$= \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}}$$

$$= \frac{2\pi\sqrt{(R+h)^2}}{\sqrt{\frac{GM}{R+h}}}$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} \text{ ------- (1)}$$

This is expression for time period for satellite.

Squaring (1)

$$T^2 = \frac{4\pi^2}{GM}(R+h)^3 \text{ --------- (2)}$$

Let R + h = r

$$T^2 = \frac{4\pi^2}{GM}.r^3$$

As, $\frac{4\pi^2}{GM}$ = constant

$$T^2 = \text{Constant} . r^3$$

$$T^2 \propto r^3$$

Thus, the square of the period o satellite is directly proportional to the cube of the radius of the orbit. This is Kepler's third law of planetary motion

Substituting $GM = gR^2$ in equation (2)

$$T^2 = \frac{2\pi^2(R+h)^3}{gR^2}$$

$$T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}} \text{ --------- (3)}$$

Substituting $GM = g_h (R+h)^2$ in equation (1)

$$T = 2\pi\sqrt{\frac{(R+h)}{g_h}} \text{ ----------- (4)}$$

EXTRA FOR COMPETETIVE EXAMS:

For a satellite orbiting very close to earth

R + h = R

Substituting in equation (2)

$$T = 2\pi\sqrt{\frac{R^3}{gR^2}}$$

$$T = 2\pi\sqrt{\frac{R}{g}} = 84 \text{ min}$$

EXPRESSION FOR THE RADIUS OF THE ORBIT:

Let 'r' be the radius of the circular orbit i.e. r = R + h

We have $T = 2\pi.\sqrt{\frac{(R+h)^3}{GM}}$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Squaring

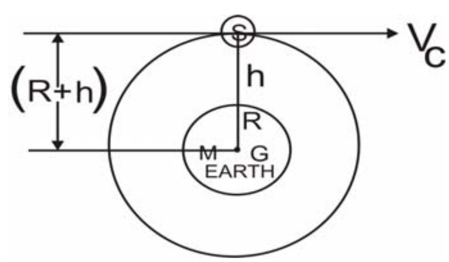
$$T^2 = \frac{4\pi^2}{GM}.r^3$$

$$r^3 = \frac{T^2 GM}{4\pi^2}$$

Taking cube root,

$$r = \left(\frac{T^2GM}{4\pi^2}\right)^{1/3}$$

Q. OBTAIN AN EXPRESSION FOR BINDING ENERGY OF THE SATELLITE ORBITING AROUND THE EARTH



Definition: ***The minimum amount of energy required or work that must be done to move a body from a point in the earth's gravitational field to infinity, against the earth's gravitational force of attraction***.

Consider a satellite of mass m orbiting around the earth with critical velocity v_c in a stable circular orbit of radius (R+h). Let M be the mass of the earth.

Expression for kinetic energy:

$$K.E. = \frac{1}{2} m v_c^2$$

Substituting $v_c = \sqrt{\frac{GM}{R+h}}$

$$E. = \frac{1}{2} m \left(\sqrt{\frac{GM}{R+h}}\right)^2$$

$$K.E. = \frac{1}{2} \frac{GMm}{(R+h)}$$

Expression for gravitational potential energy:

Gravitational potential at a point is the work done in moving a unit mass (1kg) from infinity to that point.

Gravitational potential due to mass M at a distance (R + h) from the earth's surface is,

$$V = -\frac{GM}{(R+h)}$$

Negative sign INDICATES gravitational force is the force of attraction towards the centre of mass 'M'.

Gravitational Potential Energy

= gravitational potential X mass

$$= -\frac{GM}{(R+h)}.m$$

Gravitational Potential Energy

$$= -\frac{GMm}{(R+h)}$$

Expression for total energy:

T.E. = K. E. + P. E.

$$= \frac{1}{2} \frac{GMm}{(R+h)} - \frac{GMm}{(R+h)}$$

$$= \frac{GMm}{(R+h)} \left(\frac{1}{2} - 1\right)$$

$$T.E = -\frac{1}{2} \frac{GMm}{(R+h)}$$

Negative sign for the total energy of the satellite shows that the satellite is bound to the earth by an attractive force and cannot leave. If $\frac{GMm}{2(R+h)}$ amount of energy is supplied to the satellite, the total energy satellite will become zero. Hence, the satellite will move to infinity which from definition is binding energy.

$$B.E. = \frac{1}{2} \frac{GMm}{(R+h)} \text{ -------- (1)}$$

K.E. = B.E, P.E. = -2 B.E.

Substituting GM = g R^2 in equation ------ (1)

$$\mathbf{B.E.} = \frac{\mathbf{gR^2m}}{\mathbf{2(R+h)}} \text{ ------ (2)}$$

and GM = g_h $(R+h)^2$ in equation (1)

$$B.E. = \frac{1}{2} \frac{g_h (R+h)^2 m}{(R+h)}$$

$$B.E. = \frac{g_h (R+h).m}{2} \text{ --------- (2)}$$

Q. OBTAIN AN EXPRESSION FOR BINDING ENERGY OF A BODY AT REST ON THE EARTH'S SURFACE AND ABOVE THE EARTH'S URFACE.

The minimum amount of energy required or minimum amount of work that must be done to move a body from a point in earth's gravitational field to infinity against the earth's gravitational force of attraction.

Consider a body of mass m situated at a height h above the earth's surface 'm' be the mass of the earth of radius 'R'.

As the body is at rest K. E. = 0.

Expression for gravitational potential energy:

Gravitational potential at a point is the work done in moving a unit mass (1kg) from infinity to that point.

Gravitational potential due to mass 'M' at a distance (R + h) from the earth's centre is

$$V = -\frac{GM}{(R+h)}$$

Negative sign because, gravitational force is the force of attraction towards the centre of mass ‘M’

Gravitational Potential Energy = gravitational potential x mass

$$= V.\ m$$

$$= \frac{-GM.m}{(R+h)}$$

Expression for total energy:

T. E. = K. E. + P. E.

$$= 0 - \frac{GMm}{(R+h)}$$

$$T.E = -\frac{GMm}{R+h}$$

Negative sign for the total energy of the body shows that the body is bound to the earth by an attractive force and cannot leave it.

If $\frac{GMm}{R+h}$ amount of energy is supplied to the body, the total energy will become zero.

The body will move the infinity which from definition is binding energy.

$$B.E. = \frac{GMm}{(R+h)} \text{ -------- (1)}$$

On the earth’s surface, h = 0

$$B.E. = \frac{GMm}{R} \text{ ---------- (2)}$$

Substituting $GM = gR^2$ in equation (2)

$$B.E. = \frac{gR^2.m}{R}$$

$$B.E. = mgR \text{ --------------- (3)}$$

Definition of gravitational field intensity or gravitational field strength:

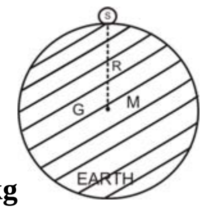
The gravitational intensity at a point in a gravitational field is the force acting on a unit mass placed at that point.

$$E = \frac{F}{m}$$

$$= \frac{GMm}{(R+h)^2.m}$$

$$E = \frac{GM}{(R+h)^2}$$

It is a vector quantity directed toward the centre of the earth.



S. I. unit = N/kg

On the earth’s surface h=0

$$g = \frac{GM}{R^2} \text{ and } E = \frac{GM}{R^2}$$

$$g = E.$$

Gravitational acceleration = Gravitational field intensity.

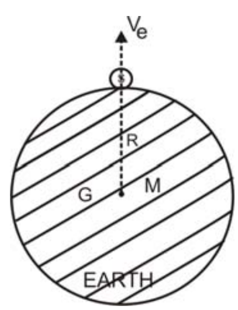
Q. Obtain an expression Escape velocity of a body at rest on the earth’s surface

OR

Prove that $V_e = \sqrt{2gR}.$

Definition: It is the minimum velocity with which a body must be projected vertically upwards, so that it escapes, the earth’s gravitational field. Such a body never returns to the earth’s surface.

EXPRESSION:



Let ‘m’ be the mass of the body which is projected upwards with an escape velocity V_e. In order to escape, if the kinetic energy given to it is equal or greater binding energy. Such a body will escape from the gravitational attraction of the earth.

Binding energy of a body at rest on the earth’s surface

$$B.E. = \frac{GMm}{R}$$

Where M is the mass of the earth R is the radius of earth.

Kinetic Energy of projection = $\frac{1}{2} m V_e^2$

Kinetic Energy of projection = B. E.

$$\frac{1}{2} m V_e^2 = \frac{GMm}{R}$$

$$V_e^2 = \frac{2GM}{R}$$

$$V_e = \sqrt{\frac{2GM}{R}} \text{ ---------- (1)}$$

This is an expression for escape velocity of body a rest on the surface

Substituting $GM = gR^2$, We get

$$V_e = \sqrt{\frac{2gR^2}{R}}$$

$$V_e = \sqrt{2gR} \text{ ------------ (2)}$$

From the above expressions, escape velocity is independent of the mass of the body.

Q. PROVE THAT THE ESCAPE VELOCITY OF A SATELLITE IN ORBITAL MOTION IS EQUAL TO CRITICAL VELOCITY. ($V_c = V_e$)

Consider a satellite orbiting around the earth with a critical velocity V_c in a stable circular orbit of radius (R + h). In order to escape, the kinetic energy given to the satellite is equal or greater than binding energy of the satellite. The satellite will escape the gravitational attraction of the earth.

Binding energy of a satellite orbiting at a height h,

$$\mathbf{B.E.} = \frac{\mathbf{GMm}}{\mathbf{2(R+h)}}$$

Where M $\rightarrow$ mass of the earth,

R $\rightarrow$ the radius of the earth.

K.E. of projection = $\frac{1}{2} m V_e^2$

Kinetic Energy of projection = Binding Energy

$$\frac{1}{2} m V_e^2 = \frac{\mathbf{GMm}}{\mathbf{2(R+h)}}$$

$$V_e^2 = \frac{\mathbf{GM}}{\mathbf{R+h}}$$

$$V_e = \sqrt{\frac{GM}{R+h}} \text{ ---------- (1)}$$

But, $$V_c = \sqrt{\frac{GM}{(R+h)}} \text{ ---------- (2)}$$

From equation (1) and equation (2)

$$V_c = V_e$$

Prove that the escape velocity of a body on the earth's surface is $\sqrt{2}$ times velocity of the body when it moves in a circular orbit close to the earth. Or Ve = $\sqrt{2}$ Vc

When a body is orbiting close to the earth's surface, radius of the orbit is equal to the radius of the earth

$$R + h \cong R$$

Centripetal force = gravitational force of attraction.

$$\frac{m.V_c^2}{R} = \frac{GMm}{R^2}$$

$$V_c = \sqrt{\frac{GM}{R}} \text{ ------------ (1)}$$

Let Ve be the escape velocity of the earth surface.

K.E. of projection = B.E.

$$\frac{1}{2} m.\ V_e^2 = \frac{GMm}{R}$$

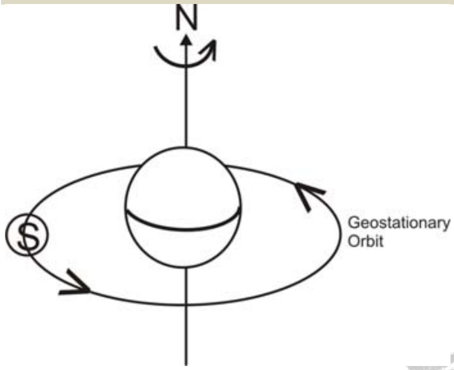
$$V_e^2 = \frac{2GM}{R}$$

$$V_e = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{2}\sqrt{\frac{GM}{R}}$$

$$V_e = \sqrt{2}\ V_c \text{ From equation (1)}$$

COMMUNICATION OR SYNCHRONOUS OR GEOSTATIONARY SATELLITE:



Those satellites orbit which once in 24 hours in the equatorial plane of the earth the same direction as that of the spin of the earth i.e. from **west to east.** Since, the period of rotations of earth about its own axis is also 24 hours, the relative velocity of the satellite w.r.t. earth is zero hence the satellite will always appear stationary above a given place from the earth's surface.

Therefore it is called geostationary satellite and the orbits are called geostationary orbit. The height of the geostationary satellite above the earth's surface is approx. **36,000 km.**

Condition for communication satellite:

1) These satellites must be in equatorial plane of the earth.
2) The direction of rotation must be same as the direction of rotation of earth in its own axis
3) Period of the satellite must be equal to the period of the earth in its own axis.
4) The height of the satellite from the earth's surface should be 36000 km

USES:
1) Used to receive and transmit radio and television signals.
2) It is also used to study earth's atmosphere and forecast weather.
3) It is used in telephone communication.

WEIGHTLESSNESS:

Feeling of weightlessness in moving satellite.

1) Gravitational attraction towards the centre of earth is by definition weight of body.
2) Weightlessness is feeling in moving satellite. It is not due to weight equal to zero.
3) When an astronaut is on surface of earth, gravitational force acts on him. This gravitational force is weight of astronaut. The earth's surface exerts an upward reaction on astronaut and due to this reaction astronaut feels his weight.
4) In an orbiting satellite astronaut along with centripetal acceleration directed towards centre of earth. Hence, astronaut can't produce any action on floor of satellite. So floor cannot produce any reaction on him.

Due to absence of this reaction astronaut has feeling of weightlessness.

NUMERICALS

1. Show that Escape velocity of a body from the surface of the earth is $\sqrt{2} \times Vc$. Where Vc is critical velocity of body when it is orbiting very close to earth surface.
2. Prove that escape velocity of a body from the surface of the planet of mean radius R and density d is $2R\sqrt{\frac{2\pi Gd}{3}}$. Where G is gravitational constant.
3. Show that critical velocity of satellite close to surface of the earth of radius R and mean density (d) is given by $2R\sqrt{\frac{\pi G d}{3}}$
4. Show that $g_h = \left[\frac{R}{R+h}\right]^2 g$ where g_h is acceleration due to gravity at a height h from the surface of the earth and g is acceleration due to gravity on the surface of the earth.

5. An artificial satellite has to be set up to revolve around the planet in circular orbit close to its surface, if ρ is mean density and R is radius of the planet show that its period of revolution is $\sqrt{\frac{3\pi}{\rho G}}$
6. Distance of the planet from the earth is 2.5 x 10^7 km and gravitational force between them is 3.82 x 10^{18} N. Mass of planet and that of earth is equal, each being 5.98 x 10^{24}. Calculate universal gravitation constant.
7. Two homogenous sphere one of mass 100 kg and other of mass 11.75 kg attracts each other with force of 19.6 x 10^{-7} N when kept with their centre 0.2 m apart. Estimate G.
8. Calculate the force of attraction between two metal spheres each of mass 90 kg if the distance between their centre is 40 cm. Given G = 6.67 x 10^{-11} Nm^2/kg^2.
9. Find the gravitational force of attraction if mass of moon is 1/81 times mass of earth. Given G = 6.67 x 10^{-11} Nm^2/kg^2. Radius of moon's orbit = 3.85 x 10^5 km M = 6 x 10^{24} kg

10. Calculate the acceleration due to gravity at the surface of the earth.
Given R = 6.4 x 10^6 m
G = 6.67 x 10^{-11} Nm^2/kg^2
Mean density ρ = 5.5 x 10^3 kg/m^3.
11. Mean radius of earth is 6400 km the acceleration due to gravity at its surface is 9.8 m/s^2. Estimate the mass of earth . Given G = 6.67 x 10^{-11} Nm^2/kg^2.
12. Find the acceleration due to gravity on the surface of moon. Given that mass of moon is $\frac{1}{80}$ times that of earth and diameter of moon is $\frac{1}{4}$ times that of earth. Given : g = 9.8 m/s^2.
13. A body weight 4.5 kg on the surface of the earth. How much it will weigh on the surface of planet whose mass is $1/9^{th}$ mass of earth and radius is half that of earth.
14. Two satellites X and Y are moving in circular orbit of radius r and 2r respectively around the same planet. What is the ratio of their critical velocity?
15. Satellite orbiting around the earth having critical velocity in the ratio 4:5. Compare their orbital radii.
16. Compare the critical speeds of two satellites if the ratio of their periods is 8:1.
17. Radius of earth is 6400 km. Calculate the velocity in km/sec with which body should be projected so as to just escape from the earth's gravitational influence.
18. What would be the speed of satellite orbiting around the earth very close to its surface Given: R = 6400km g = 9.8 m/s^2.
19. Find the height of satellite from the surface of the earth whose critical velocity is 5 km/s. By considering following data
Given: G = 6.67 x 10^{-11} Nm^2/kg^2.
R = 6400 km
M = 5.98 x 10^{24} kg
20. Find the height of satellite from the surface of the earth whose critical velocity is 4 km/s. Given :G = 6.67 x 10^{-11} Nm^2/kg^2.M = 5.98 x 10^{24} kg
R = 6.4 x 10^6 m
21. Find the height above the earth surface at which acceleration due to gravity is 50% or $\left(\frac{1}{2}\right)$ of the earth surface.
Given: R = 6.4 x 10^6 m
22. Find the height above the earth surface at which acceleration due to gravity is 90% of the earth surface.
Given: R = 6400 km
23. How far away from the centre of the earth does the acceleration due to gravity will be reduced by 1% of its value on the earth surface? Given: R = 6400 km
24. What would be the duration of a year if the distance between earth and sun gets doubled. Assuming present period o earth to be 365 days.
25. What would be the duration of a year if the distance between earth and sun gets half. Assuming present period o earth to be 365 days
26. Calculate period of revolution of planet Jupiter around the sun. Given that ratio of

radius of Jupiter's orbit to that of earth orbit 5.2: 1.

27. Communication satellite appears stationary from the place of projection. Find the distance of satellite from the surface of the earth.($G = 6.67 \times 10^{-11}$ Nm^2/kg^2, $R = 6400$ km, $M = 6 \times 10^{24}$ kg)
28. Escape velocity from the surface of the earth is 11.2 km/s. If the mass of Jupiter 3/8 times that of earth and its radius is 11.2 times that of earth. Find Escape velocity from the Jupiter surface.
29. A body is raised to a height of 1600 km above the earth surface and projected with horizontal velocity of 6 km/s. Will it remove around the earth as satellite? ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2, $R = 6400$ km, $M = 6 \times 10^{24}$ kg)
30. Satellite is taken to a height equal to radius of the earth and it is projected horizontal with speed 7 km/s. State the nature of its orbit. ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2, $R = 6400$ km $M = 5.98 \times 10^{24}$ kg)
31. A satellite is revolving in circular orbit around a planet with velocity of 8 km/sec. at a height where value of acceleration due to gravity is 8 m/sec^2. How high is the satellite from the plant's surface. Radius of planet is 6000 km.
32. An artificial satellite is revolving in circular orbit around the earth at a height of 1000 km with speed 7364 m/sec. Find the period of revolution. Given: R = 6400 km
33. Communication satellite is at a height 36000 km from the earth surface. What will be its new period when it is brought down to a height of 2000 km. Given: R = 6400 km
34. Calculate binding energy, potential energy, kinetic energy and total energy of a artificial satellite of mass 1000 kg orbiting at a height of 3600 km from the earth's surface.($M = 6 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ Nm^2/kg^2.)
35. Earth moves around the sun in circular orbit of radius 1.5×10^8 km. Calculate B.E. of the earth. (mass of sun = 2×10^{30} kg, Radius of earth (R) = 6400 km $g = 9.8$ m/sec^2.)
36. Relation between Ve and Vc for orbiting satellite
37. Satellite is orbiting around the earth with orbital radius 7000 km. If Escape velocity of satellite is 7.5×10^3 m/s. Calculate its period.

****